Position Criticality: Cyclic Zugzwangs in sub-6-man Chess

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Abstract. The zugzwang position is one in which the side obliged to move would rather not. A *Type B zug* is a win which is less deep in some sense with the loser to move. A *Cyclic Zug* is defined here as a Type B zug in which the shallower side of the initial position is critical to the win as the loser can force the line to it. A motive for finding them was that such positions can be the core of chess studies where the winner essentially has a unique route to the win. This article combines the independent searches of the first two authors for sub-6-man cyclic zugs. It also identifies some cyclic zugs in pre-existing or derived studies.

1 INTRODUCTION

Zugzwang positions, in which for some reason it is not an advantage to have the move, are relatively rare and therefore distinctive. They occur over the board and are a specific problem for chess engines. They are a focus in the endgame trilogy by Nunn (1992, 1995, 2002) and are by far the most frequently mentioned theme in 'DEM5', *Dvoretsky's Endgame Manual* (Dvoretsky, 2020; Haworth, 2021). They are an even bigger theme in the world of the composed chess study where irony is part of the artistic content. This article introduces the various types of zugzwang and two independent programmes used to identify what are defined here as, *CZ, Cyclic Zugwangs*. Full sub-6-man statistics on and examples of cyclic zugzwangs are presented: some CZs are put in the context of chess studies.

2 ZUGZWANGS OF VARIOUS TYPES

Normally, it is better to have the move in a game than not: it is better to shape the future than react to it. However, in chess, the side to move, *stm*, would sometimes prefer to 'pass', that is, to play a *null move*. *Zugzwang* or *zug* positions where this is so are very much in the minority and therefore of interest. We associate three positions with a zug: *pa*, the initial position, *pb*, the same physical position with the other side to move, and (usually redundantly) *pc*, a third position after a second null move in reply. Different reasons for not wishing to move define different types of zug:

– type A: the theoretical value of *pb* is better than that of *pa* for the first player,
– type B: *pa* and *pb* have the same value but *pb* is less deep in some metric than *pa*,
– type C: given a stochastic model of the players, the stm's expected score is better from *pb*,
– type D: (if not A, B or C) 'zug lite' (Rowson, 2005); the stm prefers to react than act.

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Type D is a matter of chessic discussion, and type C also lies beyond the scope of this article. Type A is most familiar. The first player would, with a null move, turn a draw into a win (type A1), a loss into a draw (A2) or more dramatically, a loss into a win (A3). Such positions do occur, can be missed opportunities and are more often found in sidelines by analysts, see Fig. 1.

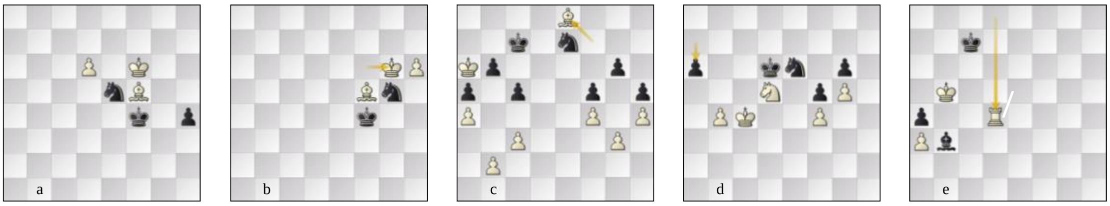

Fig. 1. (a) Lisitsyn-Zagorovsky (Leningrad, 1953), btm; (b) Fischer-Taimanov (Candidates QF g2, 1971), p87b; (c) Fischer-Taimanov (Candidates QF g4, 1971) p61b, Fischer wrong-foots the knight again; (d) Petrosian-Schmid (Bamberg, 1968), missed winning finale (a fourth weak knight), p45w (DEM5, 15-42); (e) Kasparov-Yusupov (Linares, 1993), missed win, sideline p103w (DEM5, 11-34): five type-A zugs.

With one exception, there is no point in players exchanging null moves. If a pawn can be captured en passant in position *pa*, that opportunity disappears if the null move is played instead. This scenario leads to three very rare zug types A4, A5 and A6 (Bleicher and Haworth, 2010). However, the first player would be advised not to pass in the drawn A4 situation which bounces back as a loss after a second null move. Sub-7-man, there are 394 A4, no A5, and just two A6 zugs, see Table 1.

Table 1
Some exemplar type A zugzwang positions, depth in winner's moves

#	Endgame	FEN for position *pa*	Zug		*dtc*, moves			*dtm*, moves			*dtz*, moves		
			type	**results**	***pa***	***pb***	***pc***	***pa***	***pb***	***pc***	***pa***	***pb***	***pc***
01	KPk	1k6/1P6/2K5/8/8/8/8/8 w	A1	d w d	=	2	=	=	2	=	=	2	=
02	KBkp	8/8/8/8/8/8/1pK5/kB6 w	A2	l d l	1	==	1	12	==	12	1	==	1
03	KPPkp	8/8/8/3k4/2pP4/2K5/1P6/8 b - d3	A2	l d l	1	=	1	25	=	15	1	=	1
04	KBPknp	8/8/3B2n1/K7/1pP5/k7/8/8 b - c3	A2	l d l	1	=	1	32	=	32	1	=	1
05	Kpkp	8/1pK5/kP6/8/8/8/8/8 w	A3	l w l	1	1	1	19	12	19	1	1	1
06	KRBNknn	8/8/8/8/2n5/1n6/R1N5/1B1K1k2 w	A3	l w l	1	6	1	1	16	1	1	6	1
07	KBPkpp	8/8/8/1p6/1Pp5/8/4K3/2kB4 b - b3	A4	d w l	=	3	5	=	21	20	=	1	5
08	KPPkpp	8/1p6/1k6/pP6/K7/P7/8/8 w - a6	A4	d w l	=	1	1	=	21	30	=	1	1
09	KP(5)kp(4)	8/8/8/2p5/1pP1p3/kP2P3/Pp1P4/1K6 b - c3	A5	l w d	1	1	=	19	9	=	1	1	=
10	KRPkpp	8/8/8/8/pP6/p7/k1K5/1R6 b - b3	A6	l d d	0	=	==	2	=	==	0	=	==
11	KPPPkp	8/8/8/8/1pP5/kP6/P7/K7 b - c3	A6	l d d	0	==	==	18	==	==	0	==	==

Following the convention of chess studies, we assume that it is White being challenged to achieve a result and that is to win from position *pa*. Further, the same physical position with Black to move, *pb*, is less deep. Given that the null move required to move to *pb* is a move, *pa* should be at least two ply deeper than *pb* to qualify as a type B zug. Three depth metrics are often mentioned: Depth to Conversion (DTC), to Mate (DTM) or to the Zeroing of the ply-count (DTZ). Each metric *DTx* defines its own zug type *Bx*, hence types BC, BM and BZ. In combinations, they define seven types of zug. Table 2 and Fig. 2 provide the counts of sub-6-man zugs in these seven sets. To indicate which set a type B zug is in, let a 'BxMZ zug', for example, be a BM- and BZ-zug but not a BC-zug.

A subset of type-B zugs are those positions where the loser, Black, can at least force White from *pa* to *pb*. Points are available for style and entertainment if not for the win. The line cycles back to the same physical position so these zugs are defined to be 'CZ' *Cyclic Zugs*. Such positions must be type-B zugs in any depth metric and so will be a subset of the BCMZ zugs.

Table 2.
Statistics on the seven types of sub-6-man type B zugs based on DTC, DTM and DTZ depths

#	Zugzwang		Bα zugs					Example FEN	e.p. Bμ zugs			
	Code	**Type**	**All**	**%**	**BC**	**BM**	**BZ**		**All**	**BC**	**BM**	**BZ**
1	001	**BxxZ**	68,596	14.57	——	——	68,596	8/8/8/8/pN6/8/8/k1K5 w	15	—	—	15
2	010	**BxMx**	209,496	44.50	——	209,496	——	8/2Kpk3/eR4/8/8/8/8/8 w	174	—	174	—
3	011	**BxMZ**	3,257	0.69	——	3,257	3,257	2k5/8/3K4/2P5/8/8/8/8 w	3	—	3	3
4	100	**BCxx**	50,857	10.80	50,857	——	——	1k6/8/2K5/8/8/3P4/8/8 w	77	77	—	—
5	101	**BCxZ**	31,392	6.67	31,392	——	31,392	8/1P6/8/8/8/K7/8/k7 w	8	8	—	8
6	110	**BCMx**	56,259	11.95	56,259	56,259	——	1k6/1P6/2K581P6/8/8/8 w	37	37	37	—
7	111	**BCMZ**	50,944	10.82	50,944	50,944	50,944	8/k2N4/8/p1N5/K7/8/8/8 w	32	32	32	32
		Totals	470,801	100.00	189,452	319,956	154,189		346	154	246	58

Won positions which are an essential point en route to a win are said to be *critical positions* and here include position *pb*. In chess studies, the achievement of White's challenge should be essentially unique. Thus, many or all of White's moves should be to critical positions and indeed the algorithms described below can test for this. One reason for identifying cyclic zugs is therefore that these may suggest and be the thematic focus of interesting 'win studies' in chess.

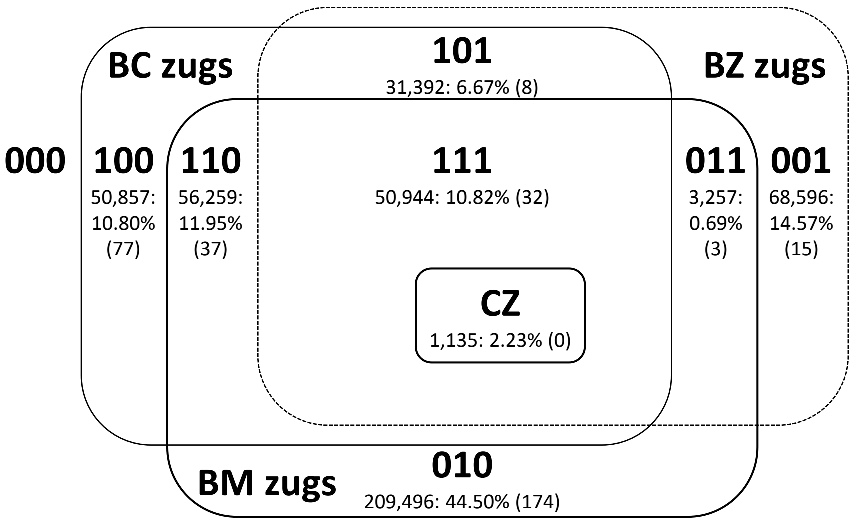

Fig. 2. The intersecting sets of sub-6-man BC, BM and BZ zugs (and 'e.p.' zugs) plus the Cyclic Zugs

The search for sub-6-man cyclic zugs was initiated by Árpád Rusz in 2013 and paused in 2015 when the limits were realised of the tools then available. Galen Huntington conducted an independent and complete search in 2020 with his improved algorithm. The next sections describe the core algorithm and the two independent programmes of Rusz and Huntington for identifying cyclic zugs.

3 POSITION CRITICALITY: ENDGAME TABLES FOR CHESS VARIANTS

To identify whether a feature of a game is essential to a win, an approach is to remove it and see if that makes a difference. For example, Bourzutschky and Konoval (2011; Konoval and Bourzutschky, 2009, 2010) removed the availability of underpromotions, first as a quick way to create 'EGT' end-game tables under the constraint P=Q, but secondly, as a way of finding situations where a P=B/N/R underpromotion was needed. Here, Z ≡ (*pa*, *pb*) is a BCMZ zug win for White. To discover whether position *pa* is a cyclic zug, i.e., whether *pb* is essential to the win from *pa*, we can create a chess variant *CV*({*pb*}) by declaring *pb* to be a draw. Is *pa* still a win in the EGT *E* for *CV*({*pb*})? If it is, *pa* is not a CZ. Otherwise, the loser can force a winning line from *pa* through *pb*. In fact, there will be a *hinterland H*(*pa*, *pb*) of positions which all lie on a *pa*-*pb* transit forced by Black.

Generalising, if we have a set of BCMZ zugs, $SB_k \equiv \{Z_i \equiv (pa_i, pb_i)\}$, there is a corresponding chess variant $CV(SB_k)$ in which all Z_i's pb_i are set to draw. In $CV(SB_k)$'s EGT E_k, if pa_i is still a win, Z_i is not a CZ. If pa_i is a draw, it may not be solely because pb_i is a draw – further investigation is needed. Let $SB_{k+1} \equiv \{Z_i \in SB_k \mid pa_i \text{ is a draw in } E_k\}$. If $SB_k \neq SB_{k+1} \neq \Phi$, we can in 'round k+1' iterate to create EGT E_{k+1} for $CV(SB_{k+1})$. At each stage, b_k BCMZ zugs are input and n_k non-cyclic BCMZ zugs are discovered. This first phase of the algorithm ends when $n_k = 0$. In phase 2, there has to be a closer focus on the remaining candidate CZs, potentially testing each individually.

Potentially, a lot of chess variants have to be investigated even though we will not have to test the 41,144 BCMZ zugs one by one. The creation of the sub-6-man EGTs can be speeded by importing some data from chess' EGT. All draws, all 0-1 wins and all 1-0 wins with a position depth less than the shallowest pb_i in S_k have the same value in chess variant $CV(S_k)$ as they do in chess.

4 THE PROGRAMME OF RUSZ AND HAWORTH

The approach (Haworth and Rusz, 2012) was to build EGTs for the endgames of assorted variants of chess as defined above.[2] We engaged keen interest and received generous help from Eiko Bleicher (2013; CPW, 2021) with his FREEZER tool and Pedro Pérez Romero (2012) with his FINALGEN tool. Both endgame generators were modified by them to preset a number of positions to 'draw' before generating EGTs for the chess variant thus defined. FREEZER created DTC EGTs and interfaced with Nalimov EGTs after force-conversion. These tools were invaluable and without them, the work would have been impracticable – but both had their limitations.

By design, FINALGEN requires that there should be at most one non-pawn piece per side.[3] This ruled it out for 15 of the 37 endgames with cyclic zugs: KQk(bn/nn/rb/rn/rr), KRk(bn/nn), KBNk(n/p), KNNkp, KQ(B/N)kq, KR(B/N)kr and KRNkn.

The implementation of FREEZER left two constraints. First, it did not handle DTC depths greater than 63 moves. Even if all chess wins in an endgame required no more than 63 moves, some might require more moves if they were to avoid a set $\{pb_i\}$ of positions.[4]

Secondly, FREEZER would not create EGTs for endgames with more than 536,864,128 positions. This was not a problem when pawns were present as these could be constrained. Also, bishops could be constrained as to square-colour which halved EGT size, as did a pair of like pieces. Even so, four of the above endgames were out of scope for FREEZER: KQkrn, KQNkq and KRNk(n/r). The full sub-6-man analysis went on pause in 2015 as it became clear that it could not be fully automated even though many CZs were identified by manual means using existing EGTs.

[2] The third author notes that Árpád did almost all the FREEZER/FINALGEN computational and investigative work.

[3] The 'one piece per side' constraint applies to most positions in Dvoretsky's (2020) manual (Haworth, 2021).

[4] This particularly affected findings in KRNkq and KNNkp where White often finds a way to bypass *pb*.

Endgame table E_1 corresponding to BCMZ-zug set SB_1 would typically reveal many non-CZs in the set SB_1 and smaller subsets SB_k were tested until no further non-CZs appeared. The main drawback of this method is that pb_i is not proved to be the sole reason for pa_i being a draw until Z_i is clearly isolated from the 'drawing effect' of all other pb_i being considered a draw.

5 THE PROGRAMME OF HUNTINGTON

Galen Huntington's independent work in 2020 confirmed that Rusz and Haworth had in fact discovered all endgames with CZs except KNPk,[5] all won CZs for 25 endgame sides, and 1,064 CZs overall.

Galen was able to create EGTs for all required s6m chess variant endgames with a modification of his own HASKELL endgame generator (Haworth, 2014a; Huntington, 2013; Huntington and Haworth, 2015). He was also able to do so more efficiently because of two new ideas.

If BCMZ zug $Z_i \equiv (pa_i, pb_i) \in SB_1$ as above, pb_i is not set to *draw* but considered a *minor win*, inferior to a *major win* that avoids pb_i but better than a draw.[6] Thus, all wins in chess variant CV_1 are *major wins* - or *minor wins* ending at pb_i if pb_i cannot be avoided. As in chess, all these wins, major or minor, are won in a finite number of ply.

The second idea is that *minor wins* are differentiated as *i-wins* by the index i of their end-position pb_i. All positions en route to an *i-win* are 'stained' with an *i-tag* including the endpoint pb_i. The positions with *i-tags* are recorded with their *i-tags* in an array T and consulted during the creation of CV_1's EGT E_1. HASKELL's rich data-structure facilities help here.

Positions, btm and even wtm, may have more than one *i-tag*. However, *i-tags* for different values i should not be considered as conjoint but rather as representing different *i-* and *j-worlds*. Huntington's labelling of *minor wins* as *i-wins*, rather than just as *minor wins*, allows us to represent separate scenarios simultaneously and focus on the individual, isolated effect of each pb_i in turn.

Galen identifies the major wins and minor *i-wins* in an EGT E_1 for chess variant CV_1. As usual, btm positions do not recognise a loss until all exits are losses. White is 'greedy' in two senses. It reaches for any win, even if a *minor win*, at the first opportunity but prefers a *major win* to a *minor win*.

Winning lines are 'grown' in the normal iterative way from 'seed' positions, i.e., backwards from their respective end-points. In addition to the normal seed-endpoint wins in 0 or 1 ply, the positions pb_i seed the minor *i-wins*. If $S_0 \equiv \{p \mid p$ is a seed win-endpoint in this endgame$\}$, S_k is defined as the set of positions given a changed value in iteration k. Winning lines k plies in length only appear after k iterations and the longest winning lines define how many iterations are required.

The introduction of the *minor win* and *i-win* concepts requires that we add to the rules for promulgating position-values as in Fig. 3. Black prefers White to achieve a *minor i-win* rather than a *major win*, see Fig. 3a. White reaches for the *i-win* of Fig. 3b but may achieve a longer *major win* later. Thus, both wtm and btm positions may gain or lose *i-tags* during this iterative process. Fig. 3c shows a position where White can avoid both an *i-win* and a *j-win* and therefore achieve a *major win*. In Fig. 3d, White can avoid a *j-win* and a *k-win* but not an *i-win*.

[5] The third author notes that he forgot about the 3-1 BCMZ zugs.
[6] Imagine that *minor wins* score 3 points and *major wins* score 4 points.

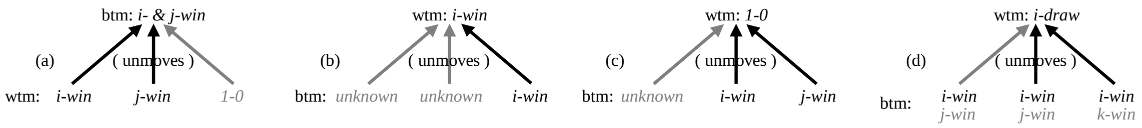

Fig. 3. Value-inheritance rules involving *minor i-win* values.

The iterative algorithm for identifying chess variant *CV's major* and *minor wins* is in summary:

– $S_0 \equiv \{p \mid p$ is the 'seed' endpoint position of a win in this endgame$\}$,
– iteration k of the algorithm gives new values to a set of positions S_k,
– iterations end when $S_{k+1} = \Phi$, the empty set: the longest win lines found are of k ply.

If pa_i is now a *minor i-win*, $Z_i \equiv (pa_i, pb_i)$ is a cyclic zug. If pa_i is not a *minor i-win*, Z_i is not a cyclic zug even if pa_i is a *minor j-win*, $j \neq i$. Only one EGT for one chess variant has been created, a clear efficiency gain.

There is a bonus here, useful given the original motivation to discover positions with study-like potential. The set of *i-win* positions may precede pa_i, indicating prior constrained White moves.

6 SUB-6-MAN CYCLIC ZUGS: STATISTICS AND EXAMPLES

Galen's statistics on sub-6-man cyclic zugs are presented in Table 3. The cyclic zugs and richer versions of the tables here are to be found in files associated with the e-version of this article.[7] Some headlines from the sub-6-man searches and results, see Tables 2 & 3 and Fig. 2:

- only 10.82% of type-B zugs are BCMZ zugs and only 2.23% of these are CZs,
- of 145 sub-6-man endgames, 59 have no BCMZ zugs and a further 50 have no CZs,
- 36 endgames have at least one CZ: only KNPkr has CZ wins for both sides,
- three endgames have over 100 CZs: KBPkn (161), KNPkr (2+128) and KPPkp (121),
- seven endgames (three 4-man and four 5-man) have just one CZ, see Fig. 4,
- 'CZ returns' from BCMZ zugs: KQkbn 0.08%, KNNkp 0.36%, KQkrr 100% (1/1),
- Philidor's (1777) KQkr position, Fig. 4, is a BCMZ zug but not a CZ,[8]
- the 1,135 Cyclic Zugs compare with 25,072 type A1 or A2 zugs (Haworth et al, 2001),
- in 2015, Árpád Rusz found 1,064 CZs and all CZ-wins for 25 endgame sides,
- none of the 32 BCMZ zugs with an e.p.-capture opportunity are CZs,
- *min∆x* ranges from 5 ply to 48 moves but …
- Black forcing a CZ-transit to *pb* can concede depth and simplify White's win,
- Assuming White minimises to some metric, at least 774 CZs have minimal transits,
- 4 KBPkp CZs are CZs in Chess-960 but unreachable in Chess,
- if the 50-move drawing rule (FIDE, 2014) applies,[9] 9 CZs are =/= and 10 CZs are =/1-0: they are all in KBNkn, KNNkp, KNNkq or KQPkq,
- passing the move, is often done by 4-move 'V, 〈 or ◊' N- or ' ∨/☐' Q-manoeuvres, by 3-move '∆' K/Q triangulation or by '(flat) ∆' Q/R/B tromboning on a line,
- CZs with other transit-manoeuvres and no Black sacrifice of depth are of special interest.

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[7] Firstly, we provide 'forced transits' under the assumption that the winner minimises to some *DTx*, *x* = C, M or Z.
[8] However, White's quickest win can start with the zug transit: **1.Qd5′ 2.Ka8″ Qa2+″ Kb8″ 3.Qa5″**. One conjecture is that the zug nature of the position and this winning line attracted both Philidor and 'Euclid' (1895) to the position.
[9] It does not apply in the world of chess studies, a specific interest here.

Table 3.
The 37 types of Cyclic Zug win in 36 endgames, with maximal *min∆x* examples.

Endgame					# BCMZ zugs			CZs		Example Cyclic Zug													
#	#m	w-b	men	GBR#	1-0	0-1	total	1-0			*dtc*			*dtm*			*dtz*			*min-*	concession ≥		
					41,067	77	41,144	1,135	%		w	b	*∆c*	w	b	*∆m*	w	b	*∆z*	*∆x*	*dtc*	*dtm*	*dtz*
01	4	2-2	KRkb	0130.00	5	0	5	1	20.00%	8/8/4R3/8/5b2/2K5/k7/8 w	14	11	3	25	22	3	14	11	3	3	0	0	0
02	4	2-2	KRkn	0103.00	43	0	43	1	2.33%	8/8/2k5/8/2K5/5R2/4n3/8 w	13	10	3	27	24	3	13	10	3	3	0	0	0
03	4	3-1	KNPk	0001.10	16	0	16	1	6.25%	1N6/k1K5/P7/8/8/8/8/8 w	10	7	3	15	12	3	9	6	3	3	0	0	0
04	5	2-3	KQkbn	1033.00	1,284	0	1,284	1	0.08%	8/8/8/8/2Kn4/k7/1b6/1Q6 w	17	6	11	28	10	18	17	6	11	11	0	7	0
05	5	2-3	KQkbp	1030.01	745	0	745	2	0.27%	8/8/7b/8/8/2Q5/4p3/1K1k4 w	14	11	3	22	19	3	14	11	3	3	0	0	0
06	5	2-3	KQknn	1006.00	2,996	0	2,996	33	1.10%	8/2n5/8/K2n4/4Q3/2k5/8/8 w	53	38	15	62	47	15	53	38	15	15	0	0	0
07	5	2-3	KQknp	1003.01	714	0	714	14	1.96%	8/8/8/8/8/1kn5/3Qp3/K7 w	14	3	11	27	21	6	14	3	11	6	5	0	5
08	5	2-3	KQkrb	1330.00	412	0	412	44	10.68%	1r6/8/8/8/1b4K1/1Q6/8/k7 w	13	5	8	41	27	14	13	5	8	8	0	6	0
09	5	2-3	KQkrn	1303.00	665	0	665	80	12.03%	8/8/8/2q5/K7/1R5N/7k/8 b	20	7	13	48	37	11	20	7	13	11	2	0	2
10	5	2-3	KQkrp	1300.01	1,974	1	1,975	87	4.41%	8/8/8/8/3Q4/1K6/4r2p/2k5 w	41	3	38	60	29	31	41	3	38	31	7	0	7
11	5	2-3	KQkrr	1600.00	1	0	1	1	100.00%	8/8/r7/8/k1KQ4/8/r7/8 w	9	6	3	30	27	3	9	6	3	3	0	0	0
12	5	2-3	KRkbn	0133.00	3	0	3	1	33.33%	8/3R4/8/4b3/8/nK6/8/k7 w	7	4	3	27	24	3	7	4	3	3	0	0	0
13	5	2-3	KRkbp	0130.01	59	0	59	17	28.81%	4R3/1b6/k1p5/2K5/8/8/8/8 w	12	7	5	32	27	5	12	7	5	5	0	0	0
14	5	2-3	KRknn	0106.00	8	0	8	5	62.50%	8/8/8/1n6/8/nK6/5R2/k7 w	6	2	4	35	32	3	6	2	4	3	1	0	1
15	5	2-3	KRknp	0103.01	844	—	844	128	15.17%	7n/2R2p2/8/8/8/4K3/8/3k4 w	27	3	24	50	25	25	25	1	24	24	0	1	0
16	5	2-3	KRkpp	0100.02	233	4	237	10	4.29%	8/8/8/8/8/1kp5/2p5/K1R5 w	6	1	5	23	20	3	6	1	5	3	2	0	2
17	5	3-2	KBNkn	0014.00	2,552	0	2,552	32	1.25%	8/8/8/8/2B5/K7/8/1kn1N3 w	54	6	48	84	27	57	54	6	48	48	0	9	0
18	5	3-2	KBNkp	0011.01	374	0	374	2	0.53%	B7/1pk5/1N6/2K5/8/8/8/8 w	7	3	4	34	31	3	6	3	3	3	1	0	0
19	5	3-2	KBPkb	0040.10	166	0	166	11	6.63%	8/8/5b2/8/5BP1/1K6/8/1k6 w	22	16	6	33	27	6	14	8	6	6	0	0	0
20	5	3-2	KBPkn	0013.10	1,169	0	1,169	161	13.77%	7n/8/8/6P1/8/8/8/kBK5 w	23	10	13	32	16	16	14	1	13	13	0	3	0
21	5	3-2	KBPkp	0010.11	1,112	0	1,112	27	2.43%	8/8/8/1BK5/7p/1k5P/8/8 w	26	19	7	40	33	7	26	19	7	7	0	0	0
22	5	3-2	KNNkp	0002.01	14,864	0	14,864	54	0.36%	8/8/2N5/8/3Kp3/1k2N3/8/8 w	61	16	45	61	16	45	58	13	45	45	0	0	0
23	5	3-2	KNPkb	0031.10	286	3	289	4	1.40%	1K6/1P2N3/1b6/2k5/8/8/8/8 w	7	4	3	23	20	3	7	4	3	3	0	0	0
24	5	3-2	KNPkn	0004.10	2,049	0	2,049	65	3.17%	3k4/1Kn5/8/P1N5/8/8/8/8 w	20	9	11	31	20	11	12	1	11	11	0	0	0
25	5	3-2	KNPkp	0001.11	3,972	0	3,972	47	1.18%	8/8/8/8/8/p1N5/1k1P4/3K4 w	13	7	6	28	22	6	8	2	6	6	0	0	0
26	5	3-2	KNPkr	0301.10	5	—	5	2	40.00%	3kr3/7P/3K2N1/8/8/8/8/8 w	7	4	3	24	17	7	7	4	3	3	0	4	0
27	5	3-2	KPPkn	0003.20	36	69	105	18	50.00%	K7/P2k4/nP6/8/8/8/8/8 w	8	5	3	18	13	5	6	3	3	3	0	2	0
28	5	3-2	KPPkp	0000.21	664	0	664	121	18.22%	8/8/8/p7/P7/1KPk4/8/8 w	19	3	16	28	17	11	9	3	6	6	10	5	0
29	5	3-2	KQBkq	4010.00	9	0	9	3	33.33%	8/1q6/k7/2Q5/8/K3B3/8/8 w	5	2	3	8	5	3	5	2	3	3	0	0	0
30	5	3-2	KQNkq	4001.00	19	0	19	4	21.05%	4q3/8/8/3Q4/k2N4/8/K7/8 w	10	7	3	12	7	5	10	7	3	3	0	2	0
31	5	3-2	KQPkq	4000.10	1,543	0	1,543	60	3.89%	8/1K1Q4/4P3/2q5/7k/8/8/8 w	37	23	14	47	33	14	29	15	14	14	0	0	0
32	5	3-2	KRBkr	0410.00	51	0	51	2	3.92%	8/8/8/8/1KB5/8/7R/2r1k3 w	20	17	3	20	17	3	20	17	3	3	0	0	0
33	5	3-2	KRNkn	0104.00	668	0	668	1	0.15%	8/8/8/8/K7/8/Nkn5/2R5 w	13	9	4	19	15	4	13	9	4	4	0	0	0
34	5	3-2	KRNkr	0401.00	18	0	18	2	11.11%	1r6/R7/8/8/8/K7/3N4/k7 w	17	14	3	24	21	3	17	14	3	3	0	0	0
35	5	3-2	KRPkb	0130.10	525	0	525	18	3.43%	8/8/1K6/3k4/3P1R2/3b4/8/8 w	41	27	14	52	36	16	31	18	13	13	1	3	0
36	5	3-2	KRPkn	0103.10	775	0	775	53	6.84%	K1k5/6R1/8/8/2n1P3/8/8/8 w	30	22	8	40	32	8	22	13	9	8	0	0	1
37	5	3-2	KRPkr	0400.10	200	0	200	20	10.00%	8/3PR3/8/3r4/1K6/3k4/8/8 w	14	8	6	28	22	6	14	8	6	6	0	0	0

7 CYCLIC ZUGS AND TRANSITS: SOME EXAMPLES

Obvious cyclic zugs to pick out are the uniques, those in studies, some demonstrating specific manoeuvres for ‘losing the move’, and those with relatively long transits. Fig. 4 (a-c, e-h) shows all the CZs that are unique in their endgame. Section 8 focuses on CZs in studies, and indeed studies composed by the second author here as inspired by CZs. In manoeuvres to ‘lose the move’, there are many different head-to-head contests and up to four men involved. Some examples:

– 3 moves, 2 men mobile, ∆/↔: K/k, Fig. 4c; K/q, $ #764; K/r $ #611; K/b $ #352;
K/n, Fig. 4b/h; Q/k, Fig. 4d/f; Q/q $ #790; Q/r #1033; Q/n $$ #333;
R/k, Fig. 4a; R/r $$ #955; R/b, Fig. 4g; B/k $$ #04; B/b; #05; B/n #06; N/k $$ #394
– 4 moves, 2 men mobile: N/k, N◊/k∆, Fig. 6e;
– 5 moves: 3 men mobile: K/kn, K↔∆/k↔n↔, Fig. 4e; KB/n, K∆B↔/k◊, Fig. 6f,
– 5 moves, 4 men mobile: KQ/kn, K↔Q∆/k↔n↔, Fig. 4i;
KR/kb, GH #966, K∆R↔/k↔b↔, Fig. 4j; KR/kn, GH #1027, K∆R↔/k↔n↔, Fig. 4k;
KB/kn, K↔B∆/k↔n↔, Fig. 4m;
– 7 moves, 4 men mobile: KR/kb, K↔↔R∆/k↔↔b↔, Fig. 4n.

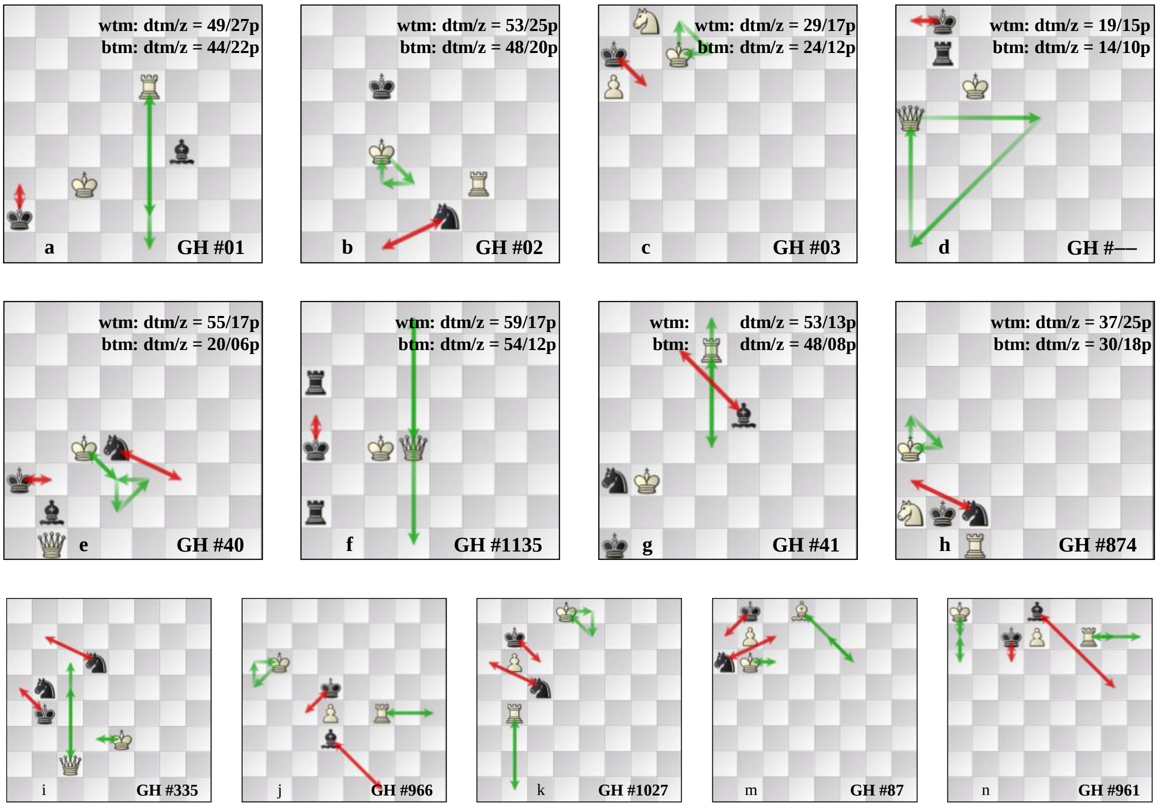

Fig. 4. Positions with 'GH' Galen indices: (a-c) The three unique four-man CZs in KRkb, KRkn and KNPk; (d) the renowned KQkr position from Philidor (1777; 'Euclid', 1895), a BCMZ zug but not a cyclic zug; (e-h) the four unique five-man CZs in KQkbn, KQkrr, KRkbn and KRNkn; (i-k, m) four 5-move CZs with four men mobile, KQ/kn, KR/kb, KR/kn, KB/kn and (n) 7-move CZ KR/kb.

Given a cyclic zug $Z \equiv (pa, pb)$, it is natural to ask about the set of forced transits from *pa* to *pb*. In this article, we assume the chess study context in which both White and Black make the best moves available. If White does not concede ground or depth, no position will be revisited, there will be no loops and so lengths of transits will be well defined. While the position-criticality algorithms described here can be used to identify time-wasting moves leading to a loop, we have not done so yet. We assume Black will maximise time in transit to the position *pb* and therefore, as a second objective, maximise depth to some metric DTC, DTM or DTZ. Certainly, if White has equi-optimal moves, there will be a choice of transits, even if there is rapid convergence downstream. Many of Black's moves can be ignored because they reduce some metric-depth to less than that of position *pb*.

Given the three depth metrics DTC, DTM and DTZ,[10] the CZ's opening and final positions have depths *dtm/c/z*(*pa*) and *dtm/c/z*(*pb*). Let $\Delta x \equiv dtx(pa)\text{-}dtx(pb) > 0$ and $min\Delta x \equiv \min(\Delta c, \Delta m, \Delta z)$. Black cannot force the transit to be longer than $min\Delta x$ though White can potentially lengthen it by vacillating en route to its win. Table 4 gives the profile of $min\Delta x$ figures. The vast majority of the 1,135 CZs exclusively feature the short, standard manoeuvres listed above. Transits are usually 5 ply as in Fig.

[10] Commonly measured in winner's moves but preferably, symmetrically and more accurately, in ply.

4.[11,12]. Table 4 shows that *min∆x* ≤ 13 ply for 93% of the CZs. Fig. 6 shows five of the CZs with longer transits at the more rarified end of the range. See Appendix 1 for some annotated lines.

Table 4.
The number of sub-6-man CZs for each *min∆x* value.

min∆x, White moves	3	4	5	6	7	8	9	10	11	12	13	14	15	16	19
no. of CZs	775	92	118	36	34	16	12	9	8	4	3	5	3	6	3
accumulated no.	775	867	985	1,021	1,055	1,071	1,083	1,092	1,100	1,104	1,107	1,112	1,115	1,121	1,124
% of CZs	68.3	76.4	86.8	90.0	93.0	94.4	95.4	96.2	96.9	97.3	97.5	98.0	98.2	98.8	99.0
no. of CZs	=1 for min∆x = 20, 21, 24, 26, 29, 31, 33, 36, 39, 45 and 48 … and 0 otherwise														

Fig. 5. Five sub-6-man cyclic zugs with longer transits: (a) KRknp 31 ply, (b) KQPkq 27 ply, (c) KRknp 27 ply, (d) KNPkn 21 ply, (e) KRPkb 21 ply. See also Fig. 6h, KNNkp, 45 ply.

8 CYCLIC ZUGZWANGS IN STUDIES

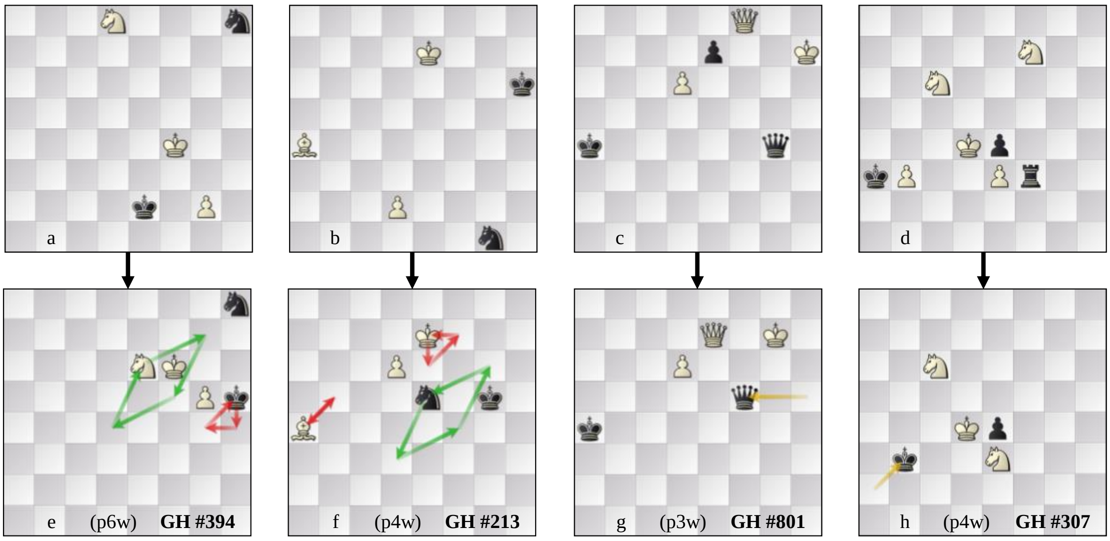

Fig. 6. (a-d) Four ‘cyclic zug studies’ by Árpád Rusz (2013) with, respectively (e-h), the CZs which they include with their ‘GH’ Galen indices:
(a/e) Magyar Sakkvilág, 2013 - CZ #394, p6w-p9b; (b/f) Jirtdan Tny, 2018 - CZ #213, p4w-p8b;
(c/g) Magyar Sakkvilág, 2013 - CZ #801, p3w-p16b; (d/h) RCS blog, 2017 – CZ #307, p4w-p48b.

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[11] Fig. 4(e): Assume White is minimising DTZ then DTM, and Black is (goal ‘*G*’) forcing the transit to *pb* and then maximising DTZ. KQkbn, S(ZM)$^-$/GZ$^+$: **1.Kd3″ Kb3″″ 2.Kd2″ Nf3+″″ 3.Ke3″ Nd4 4.Kd3″ Ka3″ 5.Kc4″**. Note that two Black moves are mandated here if Black is to force the line to *pb*. Manoeuvre descriptor: K∆↔/k,n↔.

[12] Fig. 4(h), KRNkn, S(ZM)$^-$/GZ$^+$: **1.Ka5″ Na3** (concedes depth) **2.Kb4″ Nc2+″″ 3.Ka4″** … so only 5 ply: K∆/n↔.

As stated in the introduction, an early motivation for identifying Cyclic Zugs was that these might be the basis of a chess study in which White's winning path was essentially unique. Wittingly or not, some composers have already included CZs, or at least the start or end position of a CZ, in their studies and a search by Árpád has identified some of them, see Fig. 7. Árpád has himself composed a number of studies based on CZs. Fig. 6 provides four of them and the CZ information provides clues as to their solutions which can be found in Appendix 2.

9 SUMMARY AND WAY FORWARD

The authors have continued previous work (Haworth et al, 2011a/b) on position-criticality, move-uniqueness and zugzwangs by identifying type B zugzwangs and the 'CZ' cyclic zugzwangs in sub-6-man chess. The first two authors have conducted independent programmes to identify the CZs and Huntington has identified all 1,135 of them.

Rusz identified the presence of some CZ positions in the van der Heijden (2010) study corpus. He has composed a number of studies himself in which the CZ is the main theme. Fig. 6 features four of them, including a remarkable 45-move CZ-transit without depth-concession in KNNkp. The solutions to Rusz's studies are in Appendix 2.

Datasets supporting these results are available (Huntington, Rusz and Haworth, 2021) including:

- lists of statistics and extensions of the tabular data here,
- the numbered list of Huntington's CZs with their DTC/M/Z values and Rusz equivalents,
- a list of various CZ contests of 2-, 3- and 4-men with example CZs and lines,
- a pgn of Huntington's CZs with example forced transits, and
- a list of some studies featuring one or both of a CZ's *pa* and *pb* positions in some line.

The achievements reported here further demonstrate the value of 'chess variant' EGTs for testing the criticality of downstream positions to upstream positions' values. They may be used to identify which depth-conceding moves by the attacker only result in time-wasting loops and which positions must be in the mainline solution of chess studies. Such work would further support the argument for the separation of the artistic appreciation and technical assessment of studies.

It is now possible to investigate various types of zug in 6- and 7-man chess and some CZs have already been discovered. We have identified the occurrence of cyclic zugs in some studies' mainlines and sidelines: we welcome news of others in both studies and games (Chessbase, 2021). We have not identified all the CZ transits or, in our notation of moves, eliminated all consideration of retreating moves by White which simply result in a loop. These tasks are for the future.

There are many people to thank for supporting this work over what has been nine years: the technology stack is considerable. John Beasley (2000, 2006), impressario of the mathematical and artistic in both the chess study and game worlds, has been enthusiastic about this subject from the beginning. Marc Bourzutschky has been a longtime source of EGTs, especially with his GTBGEN EGT generator and in his collaboration with Yakov Konoval on DTC EGTs. John Tamplin (2003) used Marc's GTBGEN to create and datamine DTC, DTM and DTZ Nalimov-format EGTs. Yakov provided the DTC depths here while Eiko Bleicher (2013, 2015; CPW, 2021) provided the DTM depths from Nalimov EGTs, as well as enhancing his FREEZER software for us as described. Ronald de Man (2013; CPW, 2013; Haworth, 2014b) innovated with his DTZ_{50}'' EGTs accessible via Niklas Feikas' EGT-query site. Niklas provided the DTZ values here and correlated Galen's and Árpád's findings via his index of positions in canonical form. Noam Elkies conjured the A4-A6 zugs out of his imagination. Harold van der Heijden (2010, 2020) has been evolving his HHDB database of chess studies, now in its sixth edition, and pointed to some studies featuring CZs. His continued editorship of the EG magazine on chess studies while simultaneously conducting leading edge research on SARS-CoV-2 has been a remarkable feat. Pedro Pérez Romero (2012) provided an enhanced version of his FINALGEN software (Müller and Haworth, 2019) which conveniently takes advantage of pawn placement. We also thank the referees for their comments on this article.

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APPENDICES

Appendix 1: some of the longest CZ transits[13]

The positions are indexed as in Galen Huntington's list.

CZ #519, KRknp, n7/2p5/8/5K2/7k/2R5/8/8 w, *dtm* = 42/25m, *dtz* = 33/2p.
S(ZM)$^-$/G(ZM)$^+$: 1.Rc2′ (1.Ke6′) (1.Kf6′) 1...Kh3″ 2.Ke5″ Kg4″ 3.Rc3″ Kh5″ 4.Rc1″ Kg4″ 5.Ke4″ Kg5″ 6.Rc6″ Kg4″ 7.Rc5″ Kg3″ 8.Rc2″ Kg4″ 9.Rg2+″ Kh5″ 10.Kf5″ Kh6″ 11.Rg6+″ Kh7″″ 12.Rc6″ Kg7″ 13.Ke6″″ Kh6″ 14.Kf6″ Kh5″″ 15.Kf5″ Kh4″″ 16.Rc3″ *pb*.

CZ #801, KQPkq, 8/1K1Q4/4P3/2q5/7k/8/8/8 w, *dtm* 47/33m, *dtz* 57/30p.
S(ZM)$^-$/G(ZM)$^+$: 1.Ka6″ Qa3+″ 2.Kb5′ Qb3+′ 3.Kc5′ Qc3+′ 4.Kd5″ Qa5+″ 5.Ke4′ Qe1+″ 6.Kf5″ Qa5+″″ 7.Kg6′ Qh5+′ 8.Kg7″ Qg5+″″ 9.Kf8″ Qh6+″ 10.Ke8″ Qh8+″″ 11.Ke7″ Qg7+″″ 12.Kd8″ Qf8+″″ 13.Kc7″ Qc5+″″ 14.Kb7″ *pb*.

CZ #564, KRknp, n7/2p5/8/8/4K3/6k1/2R5/8 w, *dtm* = 48/34m, *dtz* = 45/18p.
S(ZM)$^-$/G(ZM)$^+$: 1.Rc3+′ (1.Rc1′) (1.Rc4′) (1.Rc5′) (1.Rc6′) 1...Kf2″ 2.Rc1′ (2.Rc4′) (2.Rc5′) (2.Rc6′) 2...Kg2″″ 3.Kf4″ Nb6″″ 4.Rc2+″″ Kh3″″ 5.Ke4′ (5.Ke5′) 5...Nd7″″ 6.Kf5″″ Nb6″″ 7.Ke6″ Na8″″ 8.Ke5″ Kg4″″ 9.Rc3″ Kh5″″ 10.Rc1″ Kg4″″ 11.Ke4″ Kg5″″ 12.Rc6″ Kg4″″ 13.Rc5″ Kg3″″ 14.Rc2″ *pb*.

CZ #373, KNPkn, 3k4/1Kn5/8/P1N5/8/8/8/8 w, *dtm* 31/20m, *dtz* 23/2p.
S(ZM)$^-$/G(ZM)$^+$: 1.Kb8″″ Nd5″″ 2.Nd3″″ Nc7″″ 3.Ne5″ Na6+″″ 4.Ka7″ Nc5″ 5.Kb6″ Na4+″″ 6.Kb7″″ Nc5+″″ 7.Kc6″ Na6″″ 8.Kb6″ Nc7″″ 9.Kb7″″ Ne6″″ 10.Nd3″ Nc7″″ 11.Nc5″ *pb*.

CZ #965, KRPkb, 8/8/1K6/8/2kP1R2/8/6b1/8 w, *dtm* 58/47m, *dtz* 73/52p.
S(ZM)$^-$/G(ZM)$^+$: 1.Rg4″ Bh1″ 2.Kc7″ Kd5″″ 3.Rg1″ Bf3″″ 4.Rf1″″ Be2″″ 5.Rf4″″ Bc4″″ 6.Kb6′ (6. Rh4′ Bd3″″ 7.Kb6″ *p7b*) 6...Bd3″″ 7.Rh4″ *p7b* Bf1″″ 8.Ka5″ Kc4″″ 9.Rf4′ (9.Rg4′ Bh3 10.Rf4 Bg2 11.Kb6″ *pb*) 9...Bh3″ 10.Ka6″ Bg2″″ 11.Kb6″ *pb*.

CZ #61, KBPkn, k1B5/P7/K7/n7/8/8/8/8 w, *dtm* = 10/1m, *dtz* = 19/2p.
S(ZM)$^-$/G(ZM)$^+$: 1.Bd7″″ Nb7″″ 2.Kb6″″ Na5″″ 3.Be8′ (3.Ba4′) (3.Bb5′) 3...Nc4+″″ 4.Ka6″″ Na5″″ 5.B(a4/b5″) 5...Nb7″″ 6.Kb6″″ Na5′ 7.Bd7″ Nc4+′ 8.Ka6″″ Na5″″ 9.Bc8″ *pb*.

CZ #95, KBPkn, n7/8/8/1P6/8/8/8/5KBk w, *dtm* = 32/16m, *dtz* = 27/2p.
S(ZM)$^-$/G(ZM)$^+$: 1.Ba7′ (1.Bc5′) (1.Bd4′) (1.Be3′) 1...Kh2″″ 2.Kf2″″ Kh3″ 3.Kf3″″ Kh2″ 4.Be3 (4.Bd4) (4.Bc5) 4...Kh1″ 5.Kf2 Kh2″″ 6.Bf4+″ Kh1 7.Kf1″ Nb6″″ 8.Be3″ Na8 9.Bg1″ *pb*.

Appendix 2: Árpád Rusz studies - solutions

Fig. 4a/e, KNPkn, 3N3n/8/8/8/5K2/8/4k1P1/8 w with KNPkn CZ #394, p6w→p9b, a transit of 4 moves.

1.Kf5! (1.g4? Ng6+ =) **1…Kf2 2.g4 Kg3 3.g5 Kh4 4.Kf6 Kg4! 5.Ne6 Kh5**

{CZ #394, *pa***} 6.Ng7+** starts a wN ◊ diamond manoeuvre. (6.Kg7?= Ng6!) **6…Kh4′′′ 7.Nf5+ Kg4′′′** (7...Kh5 8.Ng3+ Kg4 9.Ne4 Kh5 10.Kf5 Kh4 11.Nf6 Kh3 12.Ng4 Nf7 13.g6 Nd6+ 14.Ke6 Ne8 15.Kf7 Nd6+ 16.Kf8 Nf5 17.Ne3! +-) **8.Nd4 Kh5′′′** (8...Kh4 9.Nf3+ Kg4 10.Kg7!+-) **9.Ne6′′′** {CZ #394, *pb*}

9…Ng6 (9...Kh4 10.Kg7+-) **10.Ng7+ Kg4 11.Kxg6** 1-0.

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[13] Notation. Value-preserving move-filtering strategies for White/Black: S(ZM)$^-$/G(ZM)$^+$ means "White minimises DTZ then DTM; Black moves to achieve goal G (here, to force the line to position *pb*) and then maximises DTZ and DTM." Move annotation in the context of defined move-filtering strategies: ° ≡ only legal move, ′′′′ ≡ absolutely unique move, ′′′ ≡ unique, ignoring time-wasting moves, ″ ≡ unique optimal move, ′ ≡ optimal move.

Fig. 4b/f: KBPkn, 8/4K3/7k/8/B7/8/3P4/6n1 w with KBPkn CZ #213, p4w→p8b, a transit of 5 moves.

1.d4 Nf3 (1...Ne2 2.d5 Nc3 3.d6! Nxa4 4.d7+-) **2.d5 Ne5** (2...Nd4 3.Kf6+-) **3.d6** (3.Ke6? Nc4=) **3...Kg5**

CZ #213, *pa* **4.Ke6 Nd3 5.Bb5** (5.d7? Nc5+ =) (5.Bc2? Nc5+ 6.Kd5 Nd7=) **5...Nf4+ 6.Kf7!** The thematic try. (It is too early for 6.Kd7?! position X' Ng6 7.Bd3? *(7.Ke6 Nf4+ and we are back to move 5 from the main line)* 7...Ne5+ 8.Ke6 Nxd3 9.d7 Nc5+ =) **6...Ng6 7.Ba4!** (7.d7? Ne5+ =) (7.Bc6? Ne5+ =) **7...Ne5+** (7...Kf5 8.Bc2+ +-) **8.Ke7**! CZ #213, *pb.* The 3-move wK Δ (Ke7-e6-f7-e7) plus the wB ↔ shuffle (Ba4-b5-a4) lose a move to bN's ◊ diamond (Ne5-d3-f4-g6-e5).

8…**Ng6+** (8...Kf4 9.Kf6! *(9. Ke6? Nd3 positional draw 10.Kd5 Ne5 11.Ke6 Nd3 positional draw)* 9...Ke4 10.Ke6 Nd3 11.Bc2 +- pin) **9.Ke6 Nf4+ 10.Kd7!** (position X) **Kf6** (10...Ng6 11.Bc2 Ne5+ 12.Ke6+-. Now d3 is not available for the knight. Please compare it with the thematic try.) **11.Kc8! Nd5 12.Bb3!** (12.d7? Nb6+ =) **12... Ke5!** (12...Ke6 13. d7 +-pin) **13.Kd7! Nf6+ 14. Ke7 Kf5 15. Ba2** tempo (15.Bc4) (15.Bf7) **15...Ke5 16.Be6** 1-0

Fig. 4c/g: KQPkqp, 5Q2/4p2K/3P4/8/k5q1/8/8/8 w with KQPkq CZ #801, p3w→p16b, a transit of 14 moves.

1.Qxe7! (1.dxe7? Qh5+ 2.Qh6 Qf7+ 3.Qg7 Qh5+ 4.Kg8 Qe8+ 5.Qf8 Qg6+ 6.Qg7 Qe8+ 7.Kh7 Qh5+ perpetual check – chameleon echo) **1…Qh5+!** Q-triangulation (1…Qf5+?! 2.Kg7 Kb5 3.d7 Qg4+ 4.Kh6 Qh3+ 5.Kg6 Qg4+ 6.Qg5+ +-) **2.Kg7 Qf5!**

CZ #801, *pa* **3.Kh6‴** (3.d7? Qg4+ 4.Kf8 Qf5+ 5.Ke8 Qh5+ 6.Qf7 Qe5+ 7.Kf8 Qc5+ 8.Kg8 Qg5+ 9.Kh7 Qh4+ 10.Kg7 Qg5+ 11.Qg6 Qe7+ 12.Qf7 Qg5+ perpetual check – chameleon echo) **3…Qh3+″ 4.Kg6** (minor dual 4.Kg5′ Qg3+′ 5.Kf6 *(5.Kf5)* 5…Qf3+ 6.Ke5!) **4…Qd3+ 5.Kf6 Qf3+ 6.Ke5! Qh5+ 7.Ke6** (minor dual 7.Kd4 Qd1+ 8.Kc5 Qh5+ 9.Kc6 *(9.Kb6)* 9…Qb5+ 10.Kc7) **7…Qh3+ 8.Kd5 Qd3+ 9.Kc6 Qb5+ 10.Kc7 Qa5+ 11.Kc8 Qa6+ 12.Kd8 Qa8+ 13.Kd7 Qb7+ 14.Ke8 Qc8+ 15.Kf7 Qf5+ 16.Kg7** CZ #801, *pb*

16…Kb5 17.d7 Qg4+ 18.Kh6 Qh3+ 19.Kg6 Qg4+ 20.Qg5+ 1-0.

Fig. 4d/h: KNNPPkrp, 8/5N2/2N5/8/3Kp3/kP2Pr2/8/8 w with KNNkp CZ #307, p4w→p48b, a transit of 45 moves.

$S(ZM)^{-}/G(ZM)^{+}$: **1.Nfe5 Rxe3! 2.Nc4+!** (2.Kxe3? Kxb3=) **2...Ka2!** Triangulation (If Black captures the pawn immediately, the win is much easier: 2...Kxb3 3.Nxe3 Ka3 4.Kc3 Ka4 5.Kc4 Ka3 6.Nd4 Ka4 7.Nb3 Ka3 8.Nc5 +- etc.) **3.Nxe3 Kxb3** The pawn is securely blockaded by a white knight exactly on the Troitzky line, so White must win. The win is surprisingly difficult: White can only win by reaching the same position but with Black to move! This can only be done by a 45-move-long manoeuvre!! The play is determined except some duals which can be regarded as minor ones.

CZ #307, *pa* **4.Nd8′** (minor duals 4.Ne5′ Kb4 5.Ng6 Kb5 6.Nf4) (4.Ne7′ Kb4 5.N7d5+ *(5.Ng6 Kb5 6.Nf4)* 5...Kb5 6.Ke5 *(6.Nf4)* 6...Kc6 7.Nf4) **4...Kb4″ 5.Ne6″ Kb5″ 6.Nf4″ Kc6″ 7.Ke5″ Kd7″ 8.Kd5″ Kc7″ 9.Ke6″ Kc6″ 10.Ne2!″ Kc5″ 11.Kd7″ Kb6″ 12.Kd6″ Kb5′ 13.Kd5!″** Critical position A

13…Kb6″ (13...Kb4 14.Kc6 +-) **14.Nd4″ Kb7″ 15.Ke6″ Ka6′ 16.Ne2!″ Kb7″ 17.Kd6″ Kc8″ 18.Nf4′** (minor dual 18.Nd4 Kd8 19.Nc6+ Kc8 20.Na5 Kd8 21.Nb7+ Ke8 22.Ke6 Kf8 23.Nd6 etc., reaching the main line) **18...Kd8″ 19.Ne6+′** (minor dual 19.Nh3) **19...Ke8″ 20.Ng5″ Kd8′ 21.Nf7+″ Ke8⁗ 22.Ke6″ Kf8° 23.Nd6″ Kg7′ 24.Kf5″ Kh7⁗ 25.Kf6″ Kh6⁗ 26.Ndf5+′** (minor dual 26.Nb5) **26...Kh5⁗ 27.Ng3+′** (minor dual 27.Nd4) **27...Kh4⁗ 28.Ne2!″ Kh5⁗ 29.Kf5!″** Critical position B, which looks like position A but reflected vertically!

29…Kh6⁗ The black king has to return... (29...Kh4 30.Kg6 +-) **30.Nf4″ Kg7⁗ 31.Ke6″ Kf8⁗ 32.Nh5″ Ke8⁗ 33.Ng7+″ Kd8⁗ 34.Kd6″ Kc8° 35.Ne6″ Kb7″ 36.Kc5″ Ka6″ 37.Kc6″ Ka5⁗ 38.Nc7′** (minor dual 38.Nd4) **38...Kb4″ 39.Nb5″ Ka4⁗ 40.Kc5″ Kb3″ 41.Kd4″ Kb4⁗ 42.Nc7″ Ka5′ 43.Kc5″ Ka4° 44.Ncd5″ Ka5⁗ 45.Nb4″ Ka4° 46.Nc6″ Ka3⁗ 47.Kd5!!″** Triangulation (After 47.Kd4? Kb3 we would start again with the 45-move long manoeuvre...) **47...Kb3⁗ 48.Kd4!″** CZ #307, *pb* 1-0.