# Chess : positions of reciprocal zugzwang with not more than six men 

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This document desribes a file ZUG3456.PGN which is believed to contain all chess positions with not more than six men which are reciprocal zugzwang (Black to move loses, but White to move cannot win). These can be divided into full-point reciprocal zugzwangs (whoever is to move loses) and half-point reciprocal zugzwangs (Black to move loses, White to move cannot win but can hold the draw).

The file is being distributed as a zipped file ZUG3456.ZIP of size approximately 16MB. When unzipped, this should yield a file ZUG3456.PGN of size approximately 180MB plus the present file ZUG3456README.PDF. The file ZUG3456.PGN should contain 932,789 positions.

## Presentation

The file has been designed for use and display by ChessBase and similar programs. As displayed in a ChessBase List, the White and Black men involved are shown under Players, the depths to win under Tournament, and the result with White to move in a column towards the right (so " $1 / 2-1 / 2$ " denotes a a half-point reciprocal zugzwang and " $0-1$ " a full-point). Thus the first position in the file displays as

```
Ka6 Pb7 - Kb8 DTM 9 with BTM, draw WTM 1/2-1/2
```

to indicate that the White king is on a 6 , there is a White pawn on b 7 , the Black king is on b 8 , and the depth to mate is 9 moves with Black to move. The first full-point reciprocal zugzwang in the file displays as

```
Ka7 Pb6 - Kc6 Pb7 DTM 13 with BTM, 14 WTM 0-1
```

to indicate that the depth to mate is 13 moves with Black to move and the depth to mate by Black is 14 moves with White to move, and the corresponding position with colours reversed appears as

```
Kc3 Pb2 - Ka2 Pb3 DTM 14 with BTM, 13 WTM 0-1
```

later in the file. Each full-point reciprocal zugzwang appears twice in this way unless the White men reflect the Black either on the file (three cases typified by White $\mathrm{Kb} 1, \mathrm{~Pa} 6 / \mathrm{c} 6$ against Black $\mathrm{Kb} 8, \mathrm{~Pa} 3 / \mathrm{c} 3$ ) or diametrically (again three cases, this time typified by White $\mathrm{Kg} 1, \mathrm{~Pa} / \mathrm{c} 6$ against Black $\mathrm{Kb} 8, \mathrm{Pf} 3 / \mathrm{h} 3$ ). In these six cases, flipping or turning the board and reversing the colours gives a position which is identical to the original, and so does not appear separately. For a statistical summary, see the Appendix.
The depth to win normally gives the number of White moves needed to force mate (DTM), but in the case of the five-against-one zugzwangs it gives the number of White moves needed to force any of mate, winning capture by White, losing capture by Black, or promotion (DTC). There is no significance in this distinction, which merely reflects the data that were available. Note that while DTM is necessarily positive, DTC may be zero (signifying a position where Black's only available moves are captures and each of his possible captures gives a lost ending with fewer men).

When a position is displayed by ChessBase, a single pass move " $1 .--$ " appears in the "moves" window. When an engine offered within ChessBase is used to analyse the position, the presence of this move allows the user to flip between "White to move" and "Black to move" analyses by using the left and right arrow keys.

## Sequencing and orientation

The positions are grouped by material, and are presented in a natural QRBNP sequence. Not every combination is represented, and the actual order is KP-K, KR-KB, KR-KN, KR-KP, KN-KP, KP-KB, KP-KN, KP-KP, KBP-K, KNP-K, KPP-K, KQ-KRB, KQ-KRN, KQ-KRP, and so on.

Pawnless positions are reflected so that the White king is within the triangle a8-a5-d5. If this puts it on the long diagonal, the position is further reflected so that the Black king lies on or above this diagonal; if the Black king also lies on this diagonal, the position is reflected so that the White queen lies on or above it; if the White queen is absent or also lies on it, the position is reflected so that the Black queen lies on or above it; and so on.

For positions with pawns, the squares are assumed to be ordered in sequence a7-b7-c7-...h7-a6-b6-...h2, and the position is reflected so that the White pawns are as near as possible to a7. If there are no White pawns, or if the White pawns are disposed symmetrically, the same criterion is applied to the Black pawns; if these too are disposed symmetrically, or if they are non-existent, the White king is placed in the left-hand half of the board.

Within a group of positions with the same material, the locations of the White pawns take first priority, then the locations of the Black pawns, then the White king, and so on. This means that all the positions with the same material and with the pawns in the same locations are grouped together.

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## Searching for particular positions or sets of positions

Please note that this section relates to the versions of ChessBase and CQL installed on my own machine in January 2010. There may be more recent versions offering further facilities.

Some searches for particular positions can be done using either ChessBase or CQL, but searching for positions with the men in a particular relationship to each other can be conveniently done only using CQL, and searching for positions with a particular depth to the win can be done only using ChessBase.

Positions with particular material can be picked out in ChessBase by searching on Material and in CQL by using :piececount.

Searches for pawnless positions in which one or more men are on particular squares can be performed by CQL using : flip (and :flipcolor if appropriate). This will pick up the desired position irrespective of the orientation in which it is stored in the file. ChessBase is less flexible, and to find pawnless positions with (say) the White King on e1 it is necessary to put the king on e1, h4, h5, or e8 on the 'Or' board and to use V-Mirror, or to put it on all eight squares e1/h4/h5/e8/d8/a5/a4/d1.
Searches for positions with pawns in which one or more men are on particular squares can be performed by CQL using : flipvertical and by ChessBase using V-Mirror. Searches for positions with the key men in a particular relationship to each other (for example, the "trebuchet" position with White Ka5, Pb4 against Black $\mathrm{Kc} 4, \mathrm{~Pb} 5$ ) can be performed by CQL using :flipvertical and :shift, but there is no convenient ChessBase equivalent.

Full-point zugzwangs can be found using either ChessBase or CQL by looking for positions with result 0-1.
Positions with a particular depth to mate or capture can be selected using ChessBase by specifying " $n$ " under Tournament, bracketing it by spaces so that a search for say 1 does not also pick up 10, 11, 21, etc. There is no equivalent facility in CQL.

## Note regarding legality and retro-analytical effects

The positions have been analysed ignoring legality, possible retro-analytical effects, and castling. Specifically,

- each position is presented as an object in its own right, no account being taken of whether it could have been produced by a sequence of legal moves from the normal initial array;
- no account is taken of the possibility that the side to move may have the option of an "en passant" capture;
- it is assumed that neither side can still castle, even if its king and one or both of its rooks are in their normal game array positions.

Additionally, no account is taken of the "fifty-move rule"; any eventual mate is treated as a win, even if more than fifty moves between successive captures or pawn moves necessarily occur on the way to it.

## Sourcing and validation

My immediate source was a set of spreadsheets prepared by Guy Haworth and made available on the ICGA web site. These in turn, with the exception of that relating to five-against-one positions, were prepared from data extracted from the Nalimov depth-to-mate tablebases on Eiko Bleicher's web site www.k4it. de by programs written by Marc Bourzutschky and Eiko himself. The five-against-one spreadsheet was prepared from a depth-to-capture tablebase calculated by Marc. The spreadsheets include some "en passant" zugzwangs, where the player to move is assumed to have the option of a capture en passant, but these are not included in the present file.

The statistics for zugzwangs with up to five men were checked against the well-established figures presented in a note by Guy and others in the December 2001 issue of the ICGA Journal (pages 225-230), and appear to agree exactly. The statistics for zugzwangs with six men were checked against a further spreadsheet made available by Guy on the ICGA web site, and they appear to agree exactly except in some cases where both sides have pawns. Here, Guy's statistics include the "en passant" zugzwangs which I have excluded, and direct comparison is impossible. However, a paper " 6 -man Chess and Zugzwangs" by Eiko Bleicher and Guy, presented to a subsequent ICGA conference, included a table which enabled his statistics to be adjusted to exclude these, and if these adjustments are made our results again appear to agree exactly.

Additionally, spot checks were made to verify that some expected zugzwangs did indeed appear in the file, and it was confirmed that the zugzwangs in a small random sample from the file did indeed have the properties claimed for them.

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Nearly all this work, including the verification of sample positions, depended on the validity of the Nalimov tablebases on Eiko Bleicher's web site. Some of these (for positions with up to five men, and for most pawnless positions with six men) repeated ground already covered by others, and various key parameters were verified against what was already known from other sources before they were posted. No previous data were available for six-man positions with pawns and for a time the Nalimov tablebases stood alone, but I am not aware of any respect in which they have been challenged and I think their validity is generally accepted.

## Acknowledgements

My contribution to this exercise has been entirely cosmetic, and my immediate acknowledgements are to Guy Haworth for his spreadsheets. Guy in turn makes extensive acknowledgements, citing initials which I read as referring to Eiko Bleicher, Marc Bourzutschky, Peter Karrer, Eugene Nalimov, John Tamplin, Ken Thompson, and Christoph Wirth, and I am sure that tracing back down the literature would identify further workers whose contributions, though now duplicated or overtaken, were significant advances at the time.

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## Appendix : statistical summary

The figures that follow have been pasted in from a log file produced by the program which generated the PGN file.


|  | KQ | KR | KB | KN | KP |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQ | 0 | 0 | 0 | 0 | 0 | । | 0 |
| KR | 0 | 0 | 5 | 18 | 12 | I | 35 |
| KB | 0 | 0 | 0 | 0 | 0 | । | 0 |
| KN | 0 | 0 | 0 | 0 | 22 | I | 22 |
| KP | 0 | 0 | 1 | 7 | 106 | \\| | 114 |
| al | 0 | 0 | 6 | 25 | 140 |  | 171 |

Numbers of two-against-two full-point reciprocal zugzwangs

|  | KQ | KR | KB | KN | KP | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQ | 0 | 0 | 0 | 0 | 0 | 0 |
| KR |  | 0 | 0 | 0 | 0 | 0 |
| KB |  |  | 0 | 0 | 0 | 0 |
| KN |  |  |  | 0 | 0 | 0 |
| KP |  |  |  |  | 15 | 15 |
| tal | 0 | 0 | 0 | 0 | 15 | 15 |

Numbers of three-against-one half-point reciprocal zugzwangs

| KBP | 6 |
| :---: | :---: |
| KNP | 75 |
| KPP | 43 |

Numbers of three-against-two half-point and full-point reciprocal zugzwangs (White men down, Black men across)

|  | KQ | KR | KB | KN | KP |  | Total |  | KQ | KR | KB | KN | KP | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQQ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | KQQ | 0 | 0 | 0 | 0 | 0 | 0 |
| KQR | 1 | 0 | 0 | 0 | 0 | । | 1 | KQR | 0 | 0 | 0 | 0 | 0 | 0 |
| KQB | 25 | 0 | 0 | 0 | 0 | 1 | 25 | KQB | 0 | 0 | 0 | 0 | 0 | 0 |
| KQN | 38 | 0 | 0 | 0 | 0 | , | 38 | KQN | 0 | 0 | 0 | 0 | 0 | 0 |
| KQP | 640 | 1 | 0 | 0 | 0 | 1 | 641 | KQP | 0 | 0 | 0 | 0 | 0 | 0 |
| KRR | 10 | 0 | 1 | 0 | 0 | 1 | 11 | KRR | 0 | 0 | 0 | 0 | 0 | 0 |
| KRB | 0 | 17 | 0 | 0 | 1 | 1 | 18 | KRB | 0 | 0 | 0 | 0 | 0 | 0 |
| KRN | 0 | 10 | 0 | 3 | 0 | 1 | 13 | KRN | 0 | 0 | 0 | 0 | 0 | 0 |
| KRP | 2 | 209 | 225 | 413 | 0 | । | 849 | KRP | 0 | 0 | 0 | 0 | 1 | 1 |
| KBB | 0 | 3 | 0 | 1 | 1 | । | 5 | KBB | 0 | 0 | 0 | 0 | 0 | 0 |
| KBN | 0 | 6 | 45 | 922 | 61 | । | 1034 | KBN | 0 | 0 | 0 | 0 | 0 | 0 |
| KBP | 0 | 4 | 160 | 2112 | 403 | । | 2679 | KBP | 0 | 0 | 0 | 0 | 1 | 1 |
| KNN | 0 | 0 | 0 | 362 | 3124 | । | 3486 | KNN | 0 | 0 | 0 | 0 | 0 | 0 |
| KNP | 0 | 23 | 640 | 4128 | 2281 | 1 | 7072 | KNP | 0 | 0 | 0 | 0 | 8 | 8 |
| KPP | 0 | 18 | 211 | 920 | 4179 | 1 | 5328 | KPP | 0 | 2 | 0 | 0 | 6 | 8 |
| tal | 716 | 291 | 1282 | 8861 | 10050 | 1 | 21200 | Total | 0 | 2 | 0 | 0 | 16 | 18 |

Numbers of two-against-three half-point reciprocal zugzwangs (White men down, Black men across)

|  | KQQ | KQR | KQB | KQN | KQP | KRR | KRB | KRN | KRP | KBB | KBN | KBP | KNN | KNP | KPP |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQ | 0 | 0 | 0 | 0 | 0 | 0 | 372 | 455 | 241 | 1 | 1 | 16 | 229 | 52 | 2 | I | 1369 |
| KR | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 302 | 25 | 1158 | 99 | । | 1586 |
| KB | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | । | 3 |
| KN | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 13 | 0 | 63 | 157 | । | 233 |
| KP | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 1 | 2 | 19 | 14 | 52 | । | 90 |
| tal | 0 | 0 | 0 | 0 | 0 | 0 | 372 | 455 | 243 | 1 | 4 | 333 | 273 | 1289 | 311 | \| | 3281 |

Numbers of four-against-one half-point reciprocal zugzwangs

|  | K |
| :--- | ---: |
| KBPP | 6 |
| KNPP | 93 |
| KPPP | 11 |
| ----------- |  |
| Total | 110 |

Numbers of three-against-three half-point reciprocal zugzwangs (White men down, Black men across)

|  | KQQ | KQR | KQB | KQN | KQP | KRR | KRB | KRN | KRP | KBB | KBN | KBP | KNN | KNP | KPP | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQQ | 8 | 15 | 2 | 3 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 32 |
| KQR | 0 | 236 | 1359 | 1722 | 1019 | 5 | 1 | 3 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 4346 |
| KQB | 0 | 6 | 21 | 76 | 1450 | 161 | 16 | 15 | 16 | 0 | 0 | 0 | 0 | 1 | 0 | 1762 |
| KQN | 0 | 3 | 34 | 149 | 2540 | 905 | 91 | 123 | 187 | 3 | 1 | 10 | 1 | 9 | 0 | 4056 |
| KQP | 0 | 11 | 313 | 1516 | 6652 | 1223 | 8489 | 10553 | 953 | 12 | 42 | 95 | 946 | 202 | 9 | 31016 |
| KRR | 0 | 0 | 15 | 26 | 316 | 4 | 499 | 697 | 105 | 15 | 57 | 73 | 41 | 16 | 0 | 1864 |
| KRB | 0 | 0 | 0 | 9 | 37 | 2 | 11 | 96 | 1059 | 376 | 1456 | 183 | 218 | 613 | 26 | 4086 |
| KRN | 0 | 0 | 0 | 0 | 22 | 0 | 7 | 69 | 669 | 801 | 7933 | 2113 | 8997 | 3316 | 171 | 24098 |
| KRP | 0 | 0 | 5 | 4 | 58 | 0 | 248 | 650 | 3794 | 1667 | 10807 | 12782 | 18954 | 20862 | 433 | 70264 |
| KBB | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 5 | 171 | 0 | 16 | 79 | 817 | 1507 | 72 | 2671 |
| KBN | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 7 | 251 | 0 | 16 | 1850 | 402 | 30304 | 1562 | 34394 |
| KBP | 0 | 0 | 0 | 0 | 8 | 6 | 11 | 50 | 79 | 36 | 317 | 2010 | 1368 | 16949 | 12232 | 33066 |
| KNN | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 291 | 0 | 1 | 1562 | 32 | 12426 | 26532 | 40850 |
| KNP | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 8 | 85 | 20 | 235 | 4819 | 739 | 28988 | 80091 | 114987 |
| KPP | 0 | 0 | 0 | 0 | 1 | 0 | 5 | 33 | 161 | 5 | 69 | 581 | 296 | 2482 | 32125 | 35758 |
| tal | 8 | 271 | 1749 | 3505 | 12107 | 2308 | 9384 | 12315 | 7821 | 2936 | 20950 | 26157 | 32811 | 117675 | 53253 | 403250 |

Numbers of three-against-three full-point reciprocal zugzwangs

|  | KQQ | KQR | KQB | KQN | KQP | KRR | KRB | KRN | KRP | KBB | KBN | KBP | KNN | KNP | KPP |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQQ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 0 |
| KQR |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | । | 0 |
| KQB |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | । | 0 |
| KQN |  |  |  | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |  | 2 |
| KQP |  |  |  |  | 2 | 0 | 1 | 0 | 3 | 0 | 0 | 0 | 0 | 0 | 0 | । | 6 |
| KRR |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | । | 0 |
| KRB |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | I | 0 |
| KRN |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | I | 3 |
| KRP |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 3 | 0 | 1 | 26 | । | 30 |
| KBB |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | 0 | I | 0 |
| KBN |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | 0 | 0 | I | 0 |
| KBP |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 18 | 222 | I | 240 |
| KNN |  |  |  |  |  |  |  |  |  |  |  |  | 0 | 0 | 0 | । | 0 |
| KNP |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | 144 | । | 145 |
| KPP |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 2274 | । | 2274 |
| Total | 0 | 0 | 0 | 0 | 3 | 0 | 1 | 0 | 4 | 0 | 0 | 3 | 0 | 21 | 2668 | । | 2700 |

Numbers of four-against-two half-point and full-point reciprocal zugzwangs (White men down, Black men across)

|  | KQ | KR | KB | KN | KP | Total |  | KQ | KR | KB | KN | KP | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQQQ | 0 | 0 | 0 | 0 | 0 | 10 | KQQQ | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQR | 0 | 0 | 0 | 0 | 0 | 10 | KQQR | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQB | 0 | 0 | 0 | 0 | 0 | 10 | KQQB | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQN | 0 | 0 | 0 | 0 | 0 | 10 | KQQN | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQP | 0 | 0 | 0 | 0 | 0 | 10 | KQQP | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRR | 0 | 0 | 0 | 0 | 0 | 10 | KQRR | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRB | 2 | 0 | 0 | 0 | 0 | 12 | KQRB | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRN | 2 | 0 | 0 | 0 | 0 | 12 | KQRN | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRP | 31 | 0 | 0 | 0 | 0 | 1 31 | KQRP | 0 | 0 | 0 | 0 | 0 | 0 |
| KQBB | 67 | 0 | 0 | 0 | 0 | 167 | KQBB | 0 | 0 | 0 | 0 | 0 | 0 |
| KQBN | 283 | 0 | 0 | 0 | 0 | \| 283 | KQBN | 0 | 0 | 0 | 0 | 0 | 0 |
| KQBP | 3265 | 6 | 1 | 0 | 1 | \| 3273 | KQBP | 0 | 0 | 0 | 0 | 0 | 0 |
| KQNN | 1082 | 0 | 0 | 0 | 0 | \| 1082 | KQNN | 0 | 0 | 0 | 0 | 0 | 0 |
| KQNP | 9479 | 30 | 10 | 0 | 0 | \| 9519 | KQNP | 8 | 0 | 0 | 2 | 0 | 10 |
| KQPP | 7791 | 3 | 7 | 0 | 1 | \| 7802 | KQPP | 4 | 0 | 0 | 0 | 0 | 4 |
| KRRR | 1 | 0 | 0 | 0 | 0 | 1 | KRRR | 0 | 0 | 0 | 0 | 0 | 0 |
| KRRB | 191 | 0 | 0 | 0 | 0 | \| 191 | KRRB | 0 | 0 | 0 | 0 | 0 | 0 |
| KRRN | 739 | 0 | 0 | 0 | 0 | 1739 | KRRN | 0 | 0 | 0 | 0 | 0 | 0 |
| KRRP | 3446 | 0 | 5 | 0 | 0 | \| 3451 | KRRP | - | 0 | 0 | 0 | 0 | 0 |
| KRBB | 222 | 10 | 0 | 0 | 3 | \| 235 | KRBB | 0 | 0 | 0 | 0 | 0 | 0 |
| KRBN | 983 | 87 | 2 | 0 | 3 | \| 1075 | KRBN | 0 | 0 | 0 | 0 | 0 | 0 |
| KRBP | 1827 | 1396 | 27 | 16 | 5 | \| 3271 | KRBP | 1 | 0 | 2 | 1 | 2 | 6 |
| KRNN | 198 | 84 | 6 | 0 | 5 | \| 293 | KRNN | 0 | 0 | 0 | 0 | 0 | 0 |
| KRNP | 5826 | 3933 | 212 | 24 | 1 | \| 9996 | KRNP | 2 | 0 | 0 | 10 | 0 | 12 |
| KRPP | 499 | 3344 | 74 | 128 | 5 | \| 4050 | KRPP | 1 | 2 | 0 | 2 | 0 | 5 |
| KBBB | 4 | 8 | 0 | 0 | 0 | 12 | KBBB | 0 | 0 | 0 | 0 | 0 | 0 |
| KBBN | 17 | 337 | 23 | 29 | 2 | \| 408 | KBBN | 0 | 0 | 0 | 0 | 0 | 0 |
| KBBP | 74 | 3138 | 231 | 4200 | 19 | \| 7662 | KBBP | 4 | 0 | 0 | 0 | 0 | 4 |
| KBNN | 7 | 628 | 124 | 91 | 58 | 1 908 | KBNN | 0 | 0 | 0 | 0 | 0 | 0 |
| KBNP | 315 | 21423 | 3731 | 10129 | 19 | \| 35617 | KBNP | 0 | 0 | 2 | 2 | 0 | 4 |
| KBPP | 63 | 4347 | 4242 | 17734 | 222 | \| 26608 | KBPP | 5 | 2 | 2 | 0 | 2 | 11 |
| KNNN | 1 | 82 | 1009 | 2115 | 173 | \| 3380 | KNNN | 0 | 0 | 0 | 0 | 0 | 0 |
| KNNP | 48 | 2807 | 14565 | 94991 | 511 | \| 112922 | KNNP | 0 | 0 | 0 | 4 | 0 | 4 |
| KNPP | 114 | 6317 | 27373 | 82148 | 1126 | \| 117078 | KNPP | 3 | 2 | 0 | 25 | 2 | 32 |
| KPPP | 10 | 744 | 7070 | 20995 | 3127 | \| 31946 | KPPP | 4 | 26 | 0 | 2 | 9 | 41 |
| Total | 36587 | 48724 | 58712 | 232600 | 5281 | \| 381904 | Total | 32 | 32 | 6 | 48 | 15 | 133 |

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Numbers of two-against-four half-point reciprocal zugzwangs (White men across, Black men down)

|  | KQ | KR | KB | KN | KP | Total |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| KQQQ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQR | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQB | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQN | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQQP | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRR | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRB | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRN | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQRP | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQBB | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQBN | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQBP | 6 | 0 | 0 | 0 | 0 | 6 | 6 |
| KQNN | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KQNP | 2 | 0 | 0 | 0 | 0 | 2 | 2 |
| KQPP | 12 | 0 | 0 | 0 | 1 | 13 |  |
| KRRR | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KRRB | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| KRRN | 1 | 0 | 0 | 0 | 0 | 1 | 1 |
| KRRP | 68 | 0 | 0 | 0 | 0 | 68 | 8 |
| KRBB | 22 | 0 | 0 | 0 | 0 | 22 |  |
| KRBN | 411 | 0 | 0 | 0 | 0 | 411 |  |
| KRBP | 14253 | 0 | 0 | 0 | 1 | 14254 |  |
| KRNN | 730 | 0 | 0 | 0 | 0 | 730 |  |
| KRNP | 22448 | 1 | 0 | 0 | 1 | 22450 |  |
| KRPP | 5519 | 13 | 0 | 2 | 1 | 5535 |  |
| KBBB | 463 | 0 | 0 | 0 | 0 | 463 |  |
| KBBN | 3290 | 0 | 0 | 0 | 8 | 3298 |  |
| KBBP | 5092 | 46 | 0 | 2 | 3 | 5143 |  |
| KBNN | 3779 | 0 | 0 | 0 | 17 | 3796 |  |
| KBNP | 15755 | 78 | 1 | 0 | 96 | 15930 |  |
| KBPP | 932 | 930 | 3 | 71 | 8 | 1944 |  |
| KNNN | 2886 | 1 | 0 | 0 | 6 | 2893 |  |
| KNNP | 31143 | 174 | 15 | 0 | 209 | 31541 |  |
| KNPP | 2849 | 2829 | 46 | 299 | 33 | 6056 |  |
| KPPP | 129 | 1211 | 14 | 653 | 87 | 2094 |  |
| Total | 109790 | 5283 | 79 | 1027 | 471 | 116650 |  |


| KQNPP | 58 |
| :---: | :---: |
| KRNPP | 58 |
| KRPPP | 2 |
| KBNPP | 58 |
| KBPPP | 11 |
| KNPPP | 89 |
| KPPPP | 17 |
| Total | 293 |

Total number of reciprocal zugzwangs: 929929 (927063 half-point, 2866 full-point)
Number of full-point reciprocal zugzwangs reflected into themselves on the file: 3
Number of full-point reciprocal zugzwangs reflected into themselves diametrically:
Number of positions written to output file: 932789
Full-point zugzwangs reflected neither on the file nor diametrically have been written twice

